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INVERSE PROBLEMS IN DIFFRACTION

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ABSTRACT

A two-dimensional problem of diffraction of a plane electromagnetic wave on a smooth 2π-periodic surface is considered. Numerical algorithm, solving this problem is developed.

An inverse problem of determination of the shape of 2π-periodic surface using the performance data of reverse scattering is considered.

Inverse problem was solved by means of minimization of the residual functional with the help of gradient descent method. The initial data were calculated with the help of the numerical method. On each step of iterative method of minimization ,the residual functional was calculated approximately with the help of small slope method. The examples of the shape determination are considered.

INTRODUCTION

The new approximate methods, solving the problem of diffraction on a smooth two-dimensional infinite wave-like surface (for example [1],[2]) give us hope of solving the problem of the wavy shape determination using the performance data of reverse electromagnetic scattering.

The aim of this paper is to show the advance of the approximate method of small slope in connection with the inverse diffraction problem. For this purpose we consider only periodic surfaces because for such surfaces there may be developed numerical methods, solving the direct problem with the high accuracy, so the accuracy of the approximate method, solving the inverse problem, may be investigated.

In the first part of this paper the two-dimensional direct problem of diffraction of a plane electromagnetic wave on a smooth 2π-periodic surface is considered. the numerical method, solving the direct problem of diffraction is developed:
With the help of Green's function of Flocke canal the direct problem is reduced to the one-dimensional integral equation. The kernel of the integral equation contains logarithmic singularity, which is expressed in the explicit form. The well convergent series for calculating the kernel of the integral equation are developed. The integral equation is solved using the method of moments .

In the second part of this paper the small slope method is applied for the problem of diffraction on a periodic

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surface, the algorithm, solving the inverse problem, is developed, the examples of shape determination are presented:
The inverse problem of determination of the shape of 2π-periodic surface using the performance data of reverse scattering are considered. Inverse problem was solved by means of minimization of the residual functional with the help of gradient descent method. The initial data were calculated by means of multiple solving of the integral equation . On each step of iterative method of minimization ,the residual functional was calculated approximately with the help of small slope method. The formulas for approximate calculations of residual functional are presented.

1. Mathematical formulation of the direct problem.

The unknown function u satisfies the Helmholtz equation

$$\Delta u + k^2 \cdot u = 0 \tag{1}$$

in the region $\Omega = \{ (x,y) \mid -\infty < x < f(y), \quad 0 \le y \le 2\pi \}$.
Here k is wave number, $k = \frac{\omega}{c}$, $f(y)$ is smooth 2π-periodic function.

The boundary condition for the function u is :

$$u(f(y),y) = 0 \tag{2}$$

In the region $x < x_0 = \inf\limits_{[0,2\pi]} f(y)$ the radiation condition

$$u = e^{ik(x\cdot\cos\alpha + y\cdot\sin\alpha)} + \sum_{n=-\infty}^{+\infty} T_n \cdot e^{-i\gamma_n x} e^{i\lambda_n y} \tag{3}$$

is imposed on u . Here α is the angle between the wave vector of incident wave and x - axis, $\lambda_n = k \cdot \sin\alpha + n$, $\gamma_n = \sqrt{k^2 - \lambda_n^2}$, Re $\gamma_n \ge 0$ Im $\gamma_n \ge 0$. T_n are unknown amplitudes of scattered plane waves.

The function u is also assumed to satisfy the

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Flocke conditions:

$$u(x,\ 2\pi) = u(x,\ 0)\cdot e^{it} \tag{4}$$

$$\frac{\partial u}{\partial y}(x,\ 2\pi) = \frac{\partial u}{\partial y}(x,\ 0)\cdot e^{it} \tag{5}$$

where $t = 2\pi k \cdot \sin\alpha$.

2. Mathematical formulation of the inverse problem.

Let us consider the set of direct problems (1) - (5) for resonance values of parameters k and α

$$(\alpha \in (-\tfrac{\pi}{2},0)\cup(0,\tfrac{\pi}{2}) ,\quad k = = k(\alpha) = \frac{1}{2|\sin\alpha|}). \tag{6}$$

Problem:

For the given function $T_{-\mathrm{sign}\alpha} = T_{-\mathrm{sign}\alpha}(\alpha)$ (T_n are determined in (3)) where $\alpha \in (-\frac{\pi}{2},0)\cup(0,\frac{\pi}{2})$ determine 2π-periodic function $f(y)$.

ANALYSIS

1. Numerical algorithm, solving the direct problem.

With the help of Green's function of Flocke canal

$$G(M,P) = \frac{i\pi}{2}\sum_{m=-\infty}^{+\infty} H_0^{(1)}\left[k\cdot\sqrt{\delta x^2 + (\delta y + 2\pi m)^2}\right]\cdot e^{-imt} \tag{7}$$

(here $\delta x = x_M - x_P$, $\delta y = y_M - y_P$, $H_0^{(1)}(x)$ - is the Hankel function of the first kind of order 0) the problem is reduced to the one-dimensional integral equation for the $\frac{\partial u}{\partial n}(f(y),y)$:

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$$- \frac{1}{2\pi} \cdot \int_{0}^{2\pi} G\left[f(y_M), y_M, f(y_P), y_P\right] \cdot I(y_P) \cdot \frac{\partial u}{\partial n}(f(y_P), y_P) dy_P =$$

$$= e^{ik(f(y_M)\cdot\cos\alpha + y_M \cdot \sin\alpha)} \tag{8}$$

Here $I(y) = \sqrt{1 + [f'(y)]^2}$.

The integral equation (8) was solved with the help of method-of-moments :

Let us divide the segment $[0,2\pi]$ into N equal length segments, using points y_i ($y_0=0$, $y_N=2\pi$). Consider functions :

$$\phi_i(y) = \begin{cases} 1, & y \in [y_{i-1}, y_i] \\ 0, & y \notin [y_{i-1}, y_i] \end{cases}$$

Let us seek an approximate solution of equation (8) in the following form :

$$\Psi^N(y) = \sum_{i=1}^{N} D_i^N \cdot \phi_i(y) \tag{9}$$

where coefficients D_i^N are to be determine.

Function $\Psi^N(y)$ assume to satisfy equation (8) in points $y_{i-\frac{1}{2}} = \frac{1}{2}\cdot(y_{i-1} + y_i)$. It gives following equations for D_i^N coefficients determine :

$$\sum_{i=1}^{N} D_j^N \cdot \int_{y_{j-1}}^{y_j} G\left[f(y_{i-\frac{1}{2}}), y_{i-\frac{1}{2}}, f(y_P), y_P\right] \cdot I(y_P) dy_P =$$

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$$= \exp\left\{ i k \left(f(y_{i-\frac{1}{2}}) \cdot \cos\alpha + y_{i-\frac{1}{2}} \cdot \sin\alpha \right\} \tag{10}$$

The expression (11) gives the well convergent series. which gives us the method for calculating the kernel of the integral equation (8).

$$G(M,P) = \frac{i \cdot e^{i\lambda_o \delta y} \cdot e^{i\gamma_o |\delta x|}}{2\gamma_o} +$$

$$+ \sum_{n=1}^{\infty} \left[e^{i\frac{t}{b}\delta y} \cdot \left(ch(\tfrac{t}{b}|\delta x|) \cdot \Phi_1(n,\delta x,\delta y) - \right.\right.$$

$$\left.\left. - i \cdot sh(\tfrac{t}{b}|\delta x|) \cdot \Phi_2(n,\delta x,\delta y) \right) + R(n,\delta x,\delta y) \right] \tag{11}$$

Here $M = (x_M, y_M)$, $P = (x_P, y_P)$, $b=2\pi$,

$$\Phi_1(n,\delta x,\delta y) = \frac{\cos(n\delta y)}{n} \cdot e^{-n|\delta x|} \tag{12}$$

$$\Phi_2(n,\delta x,\delta y) = \frac{\sin(n\delta y)}{n} \cdot e^{-n|\delta x|} \tag{13}$$

$$R(n,\delta x,\delta y) = \frac{i}{2} \cdot \left(\frac{e^{i\lambda_n \delta y} \cdot e^{i\gamma_n |\delta x|}}{\gamma_n} + \frac{e^{i\lambda_{-n} \delta y} \cdot e^{i\gamma_{-n} |\delta x|}}{\gamma_{-n}} \right) -$$

$$- \frac{e^{i\frac{t}{b}\delta y} \cdot e^{-n|\delta x|}}{n} \cdot \left(ch(\tfrac{t}{b}|\delta x|) \cdot \cos(n\delta y) - \right.$$

$$\left. i \cdot sh(\tfrac{t}{b}|\delta x|) \cdot \sin(n\delta y) \right) \tag{14}$$

R satisfy the expression (15)

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$$| R(n,\delta x,\delta y) | < \frac{C}{n^2} \qquad (15)$$

Rows (12), (13) can be summed:

$$\sum_{n=1}^{\infty} \Phi_1(n,\delta x,\delta y) = -\ln 2 + \frac{|\delta x|}{2} - \frac{1}{2}\cdot \ln\Big[sh^2(\frac{\delta x}{2}) + + \sin^2(\frac{\delta y}{2}) \Big] \qquad (16)$$

$$\sum_{n=1}^{\infty} \Phi_2(n,\delta x,\delta y) = \operatorname{arctg} \left[\frac{\sin(\delta y)}{e^{|\delta x|} - \cos(\delta y)} \right] \qquad (17)$$

The kernel of the integral equation contains logarithmic singularity, which is expressed in the explicit form in (16).

2. Approximate method, solving direct problem

The well known small perturbation method gives following formulas for amplitudes T_n :

$$T_n^{S.P.} = A\cdot\delta_{on} + B_n f_n + \sum_m C_{mn}\cdot f_m \, f_{n-m} \qquad (18)$$

Here $f_n = \frac{1}{2\pi}\int_0^{2\pi} e^{-iny} f(y)\, dy$, $\delta_{on} = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$

$$A = -1, \quad B_n = -2i\gamma_o, \quad C_{mn} = 2\gamma_o\gamma_m,$$

The small slope method solving the problem of diffraction on a smooth two-dimensional infinite wave-like surface was presented in [1]. In the case of periodic surface amplitudes T_n were sought in the following form:

$$T_n^{S.S.} = \frac{1}{2\pi}\int_0^{2\pi} e^{-iny}\, e^{i(\gamma_o+\gamma_n)f(y)} .$$

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$$\cdot (a_n + \sum_m b_{mn} \cdot f_m \cdot e^{imy}) dy \qquad (19)$$

where a_n, b_{mn} were constants to be determined. Let the values $T_n^{S.S.}$ satisfy the following conditions:

a) $$|T_n^{S.S.} - T_n^{S.P.}| < \text{const} \cdot \varepsilon^3 \ , \ \varepsilon \to 0 \qquad (20)$$

where $\varepsilon = \max\limits_{0 \le y \le 2\pi} |f(y)|$

b) Shift surface along x - axis condition:

$$T_n^{S.S.}[f(y)+x_o] = e^{i(\gamma_o+\gamma_n)x_o} \cdot T_n^{S.S.}[f(y)] \qquad (21)$$

c) shift surface along y - axis condition:

$$T_n^{S.S.}[f(y+y_o)] = e^{iny_o} \cdot T_n^{S.S.}[f(y)] \qquad (22)$$

Condition (20) will be valid for (18) if

$$a_o = A \qquad (23)$$

$$i(\gamma_o + \gamma_n) \cdot a_n + b_{nn} = B_n \qquad (24)$$

$$Q_{mn} = - Q_{n-m,n} \qquad (25)$$

where $Q_{mn} = C_{mn} -$

$$- \frac{\gamma_o + \gamma_n}{2} (2i \cdot b_{mn} - (\gamma_0 + \gamma_n) \cdot a_n),$$ and the

conditions (12) and (13) will in turn be satisfied if

$$b_{on} \equiv 0 \qquad (26)$$

In the article [1] the following expressions were proposed,

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which satisfy (23) - (26):

$$a_n = \frac{1}{(\gamma_n+\gamma_o)^2}(C_{on} + C_{nn} - 2i(\gamma_n + \gamma_o)B_n)$$

$$b_{mn} = \frac{-i}{2(\gamma_n+\gamma_o)}(C_{n-m,n} + C_{mn} + C_{nn} + C_{on}) - B_n$$

3. Algorithm, solving the inverse problem.

Let

$$\vec{d} = (d_1, \cdots, d_M, \cdots, d_{2M+1})$$

Consider the set of surfaces, which are determined with the help of functions:

$$f(y) = \sum_{n=1}^{m} d_n \cdot \sin ny + d_{M+1} + \sum_{n=1}^{m} d_{n+M+1} \cdot \cos ny$$

Let us suppose that we know values of the inverse scattering function for surface, corresponding to vector $\vec{d}^o$:

$$T_1^o = T(\alpha_1), \ T_2^o = T(\alpha_2), \cdots$$

$$\cdots, \ T_N^o = T(\alpha_N)$$

for angles $\alpha_1, \alpha_2, \cdots, \alpha_N$.

The numerical algorithm for vector $\vec{d}^o$ reconstruction on values $T_1^o, \cdots T_N^o$ was constructed. This algorithm is based on minimization of the residual functional with the help of gradient descent method. Direct problem on each iteration step is solved with the help of small slope approximate method. On each step of iterative method of minimization, the residual functional was calculated approximately with the help of small slope method.

Residual functional is determined by following expression:

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$$\Phi(\vec{\alpha},\vec{d}) = \sum_{m=1}^{N} | \; T_m^o - R(\alpha_m,\vec{d}) \; |^2 \qquad (27)$$

Here $\vec{\alpha} = (\alpha_1, \cdots \alpha_N)$,

$R(\alpha,\vec{d})$ is the inverse scattering functional, corresponding to small slope method:

$$R(\alpha,\vec{d}) = \sum_m \frac{1}{2\pi} \cdot \left[-\delta_{om} + i f_m \cdot \left(\frac{|ctg\, \alpha|}{2} - \gamma_m \right) \right] \cdot$$

$$\cdot \int_0^{2\pi} e^{i(m+sign\alpha)y} \cdot e^{i|ctg\alpha| f(y)} dy$$

$$f_m = \frac{1}{2\pi} \cdot \int_0^{2\pi} f(y) \cdot e^{-imy} dy \qquad ,$$

Coefficients T_m^o were calculated with the help of numerical method (7) - (17).

RESULTS

Consider the example of solving the direct problem with the help of IBM PC AT 386-387 computer. The values of parameters are : $k = 1.4$, $\alpha = 30^o$, $f(y) = 0.3 \cdot \sin y$; in numerical algorithm $N = 50$.

	Numerical algorithm	Small slope method
energy error	$0.3 \cdot 10^{-4}$	$0.16 \cdot 10^{-2}$
CPU time	3 min 54 sec	1 sec
T_{-2}	-0.072	-0.073
T_{-1}	0.349 - 0.010i	0.347 - 0.005i
T_o	-0.926 + 0.052i	-0.926 + 0.049i

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Consider examples of Furie coefficients reconstruction of surface determine function:

First simulation was carried out for N=10 , M=1 , $\vec{d}^{\,o}$ = (0, -0.3, 0.51) . After 7 iterations vector $\vec{d}^{\,*}$: (0.000, -0.300 , 0.503) was obtained as the minimum of the functional (27).

Second simulation was carried out for N=6 , M=2 , $\vec{d}^{\,o}$ = (0, 0, -0.1, 0.2 0.2) . After 44 iterations vector $\vec{d}^{\,*}$: (0.000, 0.000, -0.092, 0.195, 0.190) was obtained as the minimum of the functional (27).

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